Nature Breathes Mathematics
Mathematics as the Language of Natural Architecture





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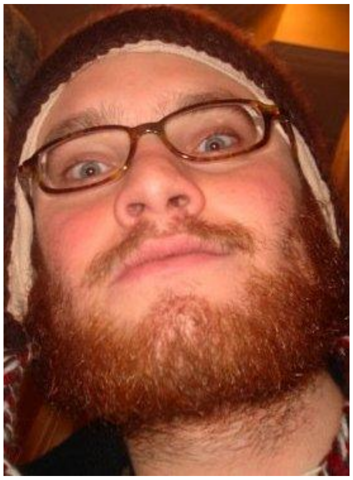
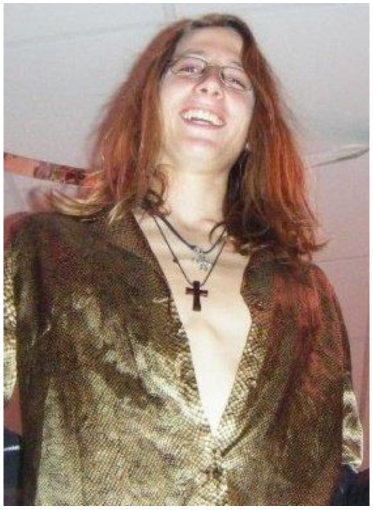
GROUP BIOGRAPHIES

Abby Jamiel is a junior Art History major from Warwick, Rhode Island. She is on the Ithaca College Varsity Crew and Snowboard teams, and she works a Tour Guide, a Research Assistant for Jennifer Germann, and a Social Coordinator for the Ithaca Honors Program. She is of half-Lebanese descent, has an older sister and a younger brother. She is possibly thinking of becoming an art conservator or owner of her own sustainable art gallery someday. She spent fall of 2009 living and traveling around Greece and hopes to return to see her friends there very soon. Summer 2010 she will be interning at the Museum of Fine Arts in Boston, Massachusetts.

Dawn Ely, a freshman at Ithaca College, is a Writing major from Pelham, New Hampshire. She is an active college student who enjoys singing with the Amani Gospel Choir, dancing with the Ithaca College Ballroom Dancing Club, doing community service with Service Saturdays, and writing short stories and poems in her free time. After graduation, she would like to find a career that incorporates both her love of creative writing and advertising. Some of her other major goals include writing and publishing at least one novel and traveling the world. She one day hopes to visit Sydney, Australia, and see the Sydney Opera House in person.



My name is **Ryan Michael Bailey**; I was born and grew up in Salt Lake City, Utah until the age of 11 when my family moved to Rochester in upstate New York. Due to the fact that I spent most of my childhood in the Rocky Mountains I naturally love to ski. My other interests include soccer, chess, economics, and movies. I will be graduating in a few weeks with honors in applied economics and in the fall will be relocating to Atlanta where I will spend the next three years of my life attending Emory Law School. This summer I plan to road trip with my brother and sister from Seattle to Madison, Wisconsin, while hoping to make it to the beach in North Carolina as many times as possible.



Greetings! My name is **Mike Moran**, I am a freshman at Ithaca College. I am a simple man who likes to get as much out of life as possible. My hobbies include playing guitar, drawing, playing soccer/lacrosse and relaxing with my friends. I'd have to say that my personality is one that is hard to find. I am laid back, fun loving, kind-hearted and try to be as peaceful and respectful as I can be. If I were to pick a life quote that sums me up it would be- Life is like a midget a urinal, always be on your toes.

ABSTRACT

Throughout the centuries and the continents of the world, mathematicians, artists, architects, and geometers alike have been awed and inspired by the power and beauty of nature. It is specifically within the realm of architecture that humans have strived to replicate the natural world. It appears a shared human condition among all people who wish to surround themselves with the environment. This paper focuses on the use of mathematics to facilitate this desire. While at times mathematics are simply the practical tools to create symmetry, shapes, and proportions, they also become the language of veneration for these very same principles. Through analyzing the Mexico Mayan temple El Castillo, the Spanish Art Nuevo cathedral of La Sagrada Familia, the early 20th century American designs of Frank Lloyd Wright, and the

modern construction of the Australian Sydney Opera House, this paper uncovers many cultural universals. In all these architectural instances, mathematics are used to highlight and enhance the beauty of the natural world. Constructing mathematics becomes the language that transcends cultural boundaries, allowing both humans and nature to express themselves.

INTRODUCTION

Math and nature are fundamental aspects of architecture, whether it is cutting uniform two by fours or clearing a lot to make way for a new home. Nature and math are not necessarily a burden to the architect or a nuisance that comes with the job, however. Many architects have found ingenious ways to incorporate elements of nature, such as the materials used and the location of the structure, into their creations. At the same time, these architects used aspects of geometry and other realms of mathematics in such ways that they enhanced the natural appearance and feel of the building. The implicit and explicit relationships that exist between mathematics and nature can be applied to architecture from all over the world and in all different time periods. As a group, we looked across cultures and time periods at specific examples of architects who took a holistic, organic approach to creating buildings. These architects also use similar mathematics in their designs, such as fractal geometry.

For this project, Mike chose to examine “The Castle” in Mexico because in one way or another most Americans are familiar with Mayan culture and in particular their incredibly unique, still standing temples. “The Castle” of El Castillo is a Mayan temple built between the 10th and 13th centuries in Chichen Itza, Mexico on the Yucatan peninsula. The temple pays homage to a serpent deity, which is made clear by the serpents that run the length of the stairs. Mike also worked on the conclusion paragraph for the paper.

Dawn decided to examine the Sydney Opera House in Sydney, Australia, as her building. Designed in the 1950s, this modern-day architectural wonder has become the symbol of Sydney and a home to the city's first-ever music performance center. With inspiration from Mayan architecture to reach to the heavens and replicate the natural world, architect Jorn Utzon created the famous white shells that today are recognizable to people worldwide. Dawn also worked on the homework section and edited the paper.

Having recently returned from a trip to Europe, Abby chose to examine the temple of Sagrada Familia in Barcelona, Spain. This magnificent church is the masterpiece of the famed organic architect Antoni Gaudi. Nature is embraced whole-heartedly in this structure through the use of complex but naturally appearing shapes as well as explicit representations of nature such as the columns made to look like trees. Abby also wrote the title page, the table of contents, and the abstract for the paper.

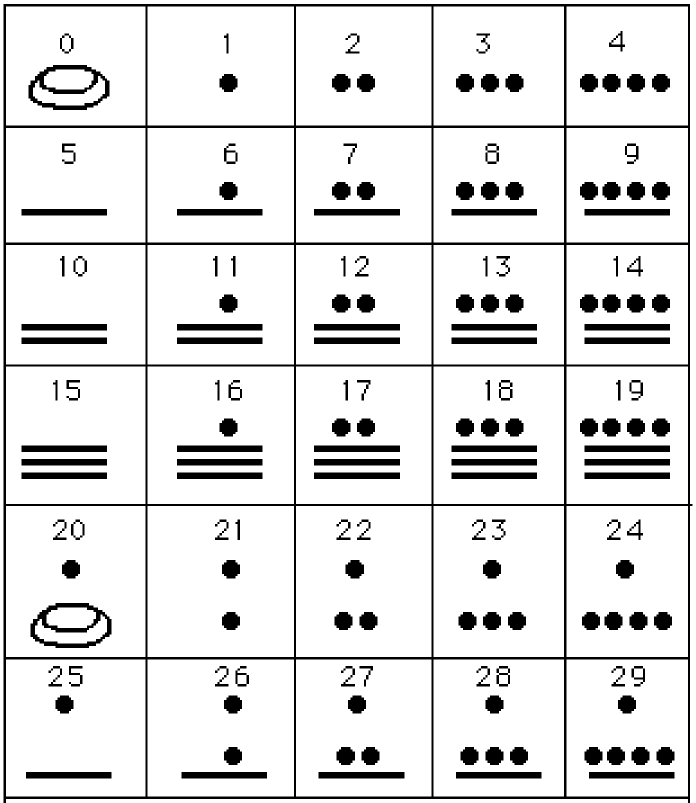
Ryan's focus for the project consisted of two buildings, a house and a civic center, both designed by the famed American architect Frank Lloyd Wright. The first building, the Marin County Civic Center in San Rafael, California was built over 50 years ago, but provides an incredible visualization of fractal geometry used in the creation of an entire façade of a building. It also embodies within its shape and construction the principles Wright lived by, which were that architecture should adhere to its surrounding, not the other way around and that buildings should be one with nature and organic. The second building is the Palmer house in Ann Arbor, Michigan, another spectacle of fractal use but this time in domestic architecture. Ryan also wrote the introduction for the paper.

What we hope to achieve by viewing these four completely distinct styles of architecture is to show the universality of the idea that architects use mathematics both to replicate and venerate the natural world. Even though these buildings are set in very different time periods and cultures, the concept that there is some intrinsic linkage between mathematics and nature transcends all boundaries, and this idea comes to fruiting through architecture because of its close relationship with both math and nature.

MAYAN MEXICO: EL CASTILLO

To tackle the issue of showing how natural mathematical designs and ideas have transcended time, culture, and religion, I opted to start with a significantly older building. I chose El Castillo because in one way or another most Americans are familiar with the Mayan culture and in particular their incredibly unique, still-standing temples. “The Castle” or El Castillo - The Temple of Kukulkan, as it is called, is a Mayan temple that was built between the 10th and 13th centuries in Chichen Itza, Mexico on the Yucatan peninsula. The temple pays homage to the feathered serpent deity, Kukulkan, which is made clear by the two serpents that run parallel the length of one of the sets of stairs.

The Maya are probably the best known of the classical civilizations of Mesoamerica. The history of the Mayan culture starts in the Yucatan peninsula around 2600 B.C, although Mayan history only really rose to prominence around A.D 250. Mexico, Western Honduras, El Salvador, and Northern Belize are a few of the many sites of Mayan civilization. The Mayans have been hailed for their precise and elaborate time-keeping devices. Building on the inherited inventions and ideas of earlier civilizations such as the Olmec, the Mayans developed astronomy, calendrical systems and hieroglyphic writing. The Mayans were noted as well for elaborate and highly decorated ceremonial architecture, including temple-pyramids, palaces and observatories, all built without metal tools. Their culture placed immense value in astronomy and the movement of the stars, moon, and the cosmos. Because of this, their cities and/or urban centers were essentially timepieces, with temples such as El Castillo sitting at the center, serving civic, religious, and agricultural purposes. To achieve this, they measured the rising and setting sun during the solstices and equinoxes, as well as its passing through the zenith. They recorded the cardinal points and data related to other stars, the Moon, planets (mainly Venus) and the constellations. There was a strong religious and astrological connection to the building endeavors of the Mayans. For example, they might build a temple in a specific location so that if you faced the front you would be looking south to see the path of the Jaguar. The sun, rising in the east and setting in the west, would circle the temple. The steps to the temple would be placed exactly in the middle so that the priest would be in better touch with the gods.



Mayan positional number system

The Maya number system was a base twenty system. Almost certainly the reason for base twenty arose from ancient people who counted on both their fingers and their toes. Although it was a base twenty system, called a vigesimal system, one can see how five plays a major role, again clearly relating to fingers and toes. In fact, it is worth noting that although the system is base twenty, it only has three number symbols (perhaps the unit symbol arising from a pebble and the line symbol from a stick used in counting). They used numbers as well as data taken from the observation of the sun, stars, moon, and planets and there relativity in order to create such precise timepieces and calendars.

El Castillo as a structure reaches seventy-nine feet high, just above the level of the forest (it is easy to imagine that these temples were also used for defensive purposes, as well as portraying might). Its dimensions on the temple create a square with sides of 179 feet. The basic structure is in the shape of a terraced pyramid and each side has a staircase comprised of ninety-one steps. The sum of all the sides as well as the top platform comes to 365, dedicating each step to one day of the year. What is probably the most fascinating architectural element of El Castillo is the fact that it is positioned in such as way that at each of the equinoxes (spring and fall), the sun will hit one of the sides so minimally that the light creates the apparition of the body of a snake or serpent. The sun casts seven triangular shadows over the left corner of the pyramid, forming a shade that appears to be the body of a serpent exiting the top on the temple and slithering down to the bottom of the stair set where the sculpture of the serpent's head is located. The snake consists of seven triangles that last for a flittering moment, representing one for each day of the week.

The geometry of the structure looking straight on at a single side represents an isosceles trapezoid; from above, however, it is the shape of a square with the staircase structure creating a type of fractal appearance. In the overhead view, we see that the pyramid has four sides, and four staircases. We now have all the numbers shown by the architect and can put the decoding formula together:

9 terraces x 365 steps x 4 sides x 4 stairways = 52,560

There are also other numbers that also multiply to 52,560. These are 119, 42 and 10.51620648. There are two rational numbers and an irrational, but these are not shown on the pyramid; these appear only on maps. When the world's pyramids were built, their longitudes were reckoned from a very ancient Prime Meridian (0/360° longitude) that ran from pole to pole across the Great Pyramid at Giza, a full 31 degrees, 08 minutes, 00.8 seconds to the east or our modern Greenwich Prime Meridian. In order to "read" our western pyramids, this 31° 08' 00.8" longitudinal variance must be factored in to our present-day longitudes for these Western monuments. Those other three numbers of 119, 42, and 10.51620648 shown above, are the elements of Kukulkan original longitude that multiplies to its GRID longitude, which was left to us in the 4-4-9-365 messages, conveyed by the Kukulkan itself.

The mathematics present in El Castillo show the intelligence of the Mayans in trying to display nature by using numbers. From the use of the sun to display a snake on the side of the temple to the creation of their own number system, the Mayans were equally skills as mathematicians and natural observers. Their famous pyramids have, and continue to, inspired generations of architects to reach for the sky and beyond in their own buildings.

JORN UTZON: SYDNEY OPERA HOUSE

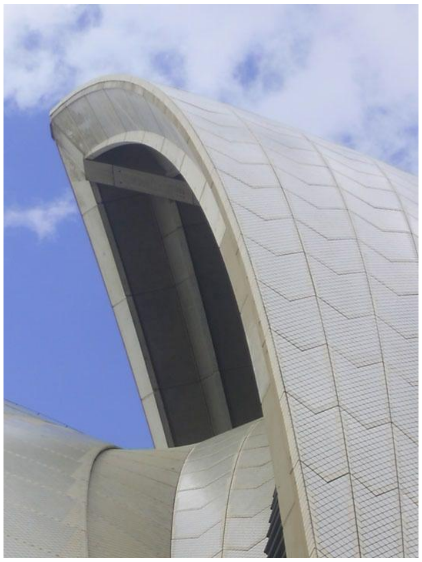
Inspired by many natural designs including Chichen Itza in Mexico, the Sydney Opera House has become the symbol of the well-known city of Sydney, Australia. An architectural wonder of its time, the opera house amazes visitors year-round with its magnificent white roof vaults and unique design. While the construction is based around simple principles of geometry, its design took architect Jorn Utzon many years to conceive and has since paved the way for similar construction principles used in other buildings worldwide. The union of nature and mathematics can be found in almost every element of the structure, drawing more and more people each year to Sydney in order to view this magnificent artistic wonder.

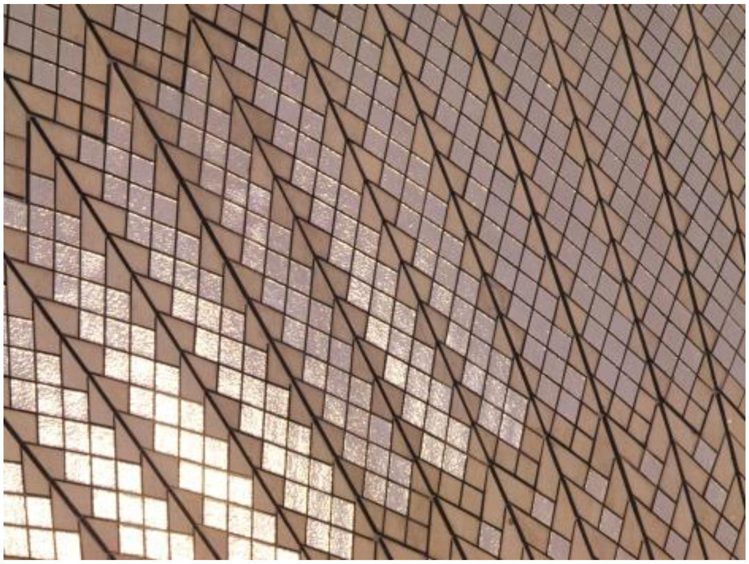
Located at the tip of Bennelong Point in Sydney Harbor, the Sydney Opera House is the busiest performing arts center in the world. Since it opened in 1973, it has hosted thousands of theatre performances, musicals, operas, dance recitals, symphony concerts, exhibitions and films, averaging around three thousand events held yearly. The building welcomes about two million visitors per year, and 200,000 guests annually take tours of some of the opera house’s one thousands rooms, including its five theaters, five rehearsal studios and numerous food and souvenir sites. It is open every day of the year except for Good Friday and Christmas, allowing the 611 feet long and 380 feet wide building to get plenty of use as tourists from all over the world come to walk the halls of this declared World Heritage Site ("Sydney Opera House…").

Before the Sydney Opera House was constructed, the city of Sydney had no official place to hold musical events for public viewing. In 1947, the idea to create a concert hall that could hold both operas and orchestra concerts emerged, and an international design contest was held to find an architect for the project. Out of 233 entries, the plans of Jorn Utzon from Denmark caught the attention of the opera house committee due to its unique sail-like roof design, and Utzon was hired in 1957 to begin construction on the building. While the construction was estimated to take four years to complete, complications faced by Utzon and the other architects moved the completion date back by ten years. Utzon was even forced to resign in the middle of the project because of budget cuts and pressures from newly elected government members. He left Australia soon after his resignation and was never able to return to Sydney to see the completion of the opera house ("Sydney Opera House…"). A few years before he died, Utzon received architecture's highest award, the Pritzker Prize, for his design of the Sydney Opera House ("The Sydney Opera…"). Today Jorn Utzon is honored as the architectural genius who created the masterpiece that is the opera house, and a room of the Sydney Opera House is dedicated to his memory.

In designing the opera house, Utzon was inspired not only by previous architects but also by the natural world around him. In 1949, Utzon was awarded a scholarship to tour the United States and Mexico to study architecture. On this trip he was able to meet with the famous architect Franklin Lloyd Wright, who influenced the use of nature in his designs, and see the Mayan pyramid Chichen Itza in Mexico, which inspired him to create buildings like the opera house that were directed upwards towards the sky ("Jorn Utzon"). Nature also played a large role in the construction of the opera house, and Utzon incorporated many natural elements into the opera house's design. The roof vaults, or shells, of the building are white to reflect sunlight and remind visitors of the pure colors of nature as seen in clouds and onions. As the American architect Louis Kahn observed, "The sun did not know how beautiful its light was until it was reflected off this building" (Utzon 58). Utzon was also inspired by the structure of leaves, the entrance to Sydney Harbor and the folds of birds' wings, using these ideas in the construction of the building's structural supports (Utzon 58). Visiting famous architects and natural wonders helped Utzon to create the masterpiece that is the Sydney Opera House.

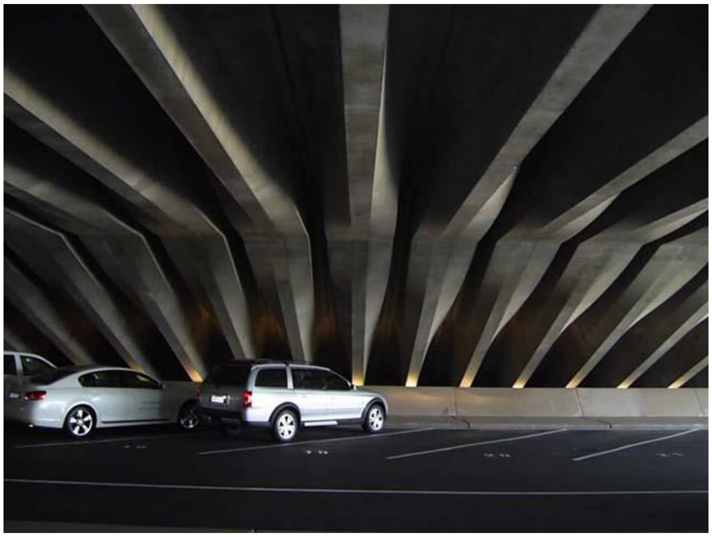
When Jorn Utzon first began to design the opera house, he encountered most of his design problems with the most prominent and recognizable feature of the building: the roof vaults. With his team of architects by his side, Utzon spent four years experimenting with paraboloid, ellipsoid, parabolic and circular shapes to design the shell ridges (the center point of the shell where the two symmetrical, curvilinear-triangle-shaped sides meet). None of these shapes worked, however, because the materials used to create the roof vaults (the tiles, the wood, etc.) wouldn't be able to mass produce easily due to the unusual shapes (Hammer 49). Finally, in 1961, Utzon came up with the ideal design solution: cut all of the shells from the surface of a sphere. As the story goes, Utzon was dismantling a model of the opera house in his shop and was stacking the roof shells inside one another. When he looked at the shells, he noticed how similar their curvatures were to one another. Because of this, he reasoned they could all be cut from a common surface that had the same curvature in all directions: a sphere (Bentley). In thinking about his design many years later, Utzon noted that he was inspired by the segmented shape of an orange for the roof, not white ship sails like some rumors suggest. "Think of a sphere or a globe – or an orange. Now slice it into quarters, or any number of adjoining shapes. All have the same radius, simplifying construction. These slices form the roofs: an elegant and simple solution" ("Sydney Opera House"). Designing the roof of the opera house was the greatest challenge for Utzon, but his hard work paid off in the beautiful and unique design that now shapes the Sydney Opera House.





In order to give the roof vaults their white color, 1,056,056 Swedish ceramic tiles were placed on 4,253 concrete lids that cover the wooden ribs which form the inside of the shells ("Sydney Opera House…"). Two different colored tiles, one a glossy off-white and the other a matte cream, were positioned on the roof vaults to create a pattern that would reflect the sunlight (Hammer 51). Each of the three main tile patterns displayed on the shells possesses vertical symmetry in addition to the first type of frieze pattern (a pattern that has only traditional symmetry in one direction) since only translations are used. These interesting pattern features contribute to the opera house's unique design and structure.

Another interesting aspect of the Sydney Opera House's design can be found in the glass walls that face Sydney Harbor. There are twenty four glass walls located in the opera house, and the two main walls in the front of the opera house's largest theaters are placed size by side diagonally so visitors can admire a panoramic view of the water. What is most fascinating about these two walls is that they are composed of two shapes: an elliptical cylinder and a cone. The top, vertical part of the windows resemble the elliptical cylinder shape, while the middle and the bottom parts of the wall are shaped like the bottom of a regular cone (Hammer 51). Using these shapes in the design of the glass walls helped to facilitate its construction and add beauty to the building.



Even though the parking garage for the Sydney Opera House is not a part of the opera house itself, the design of this structure, known as a vehicle concourse in Australia, employs designs credited as the first of their kind in the world. Created by Ove Arup, the consulting engineer and administrator of the opera house, the vehicle concourse houses cars in a double helix-shaped building and has fifty-two concrete beams running along the ceiling of the structure. These beams are quite remarkable in construction, as no supporting columns are needed over the fifty square meter area where the cars are parked. The beams also resemble a sinusoidal curve. According to experts, this same design is also used in the Sydney Opera House's shells for the ribs that support the tiles (Hammer 52). The vehicle concourse for the opera house is another remarkable feat in both architecture and mathematics.

The Sydney Opera House is one of the most fascinating buildings of study for architects and artists alike, drawing visitors from all over the globe to view and tour its remarkable design. Jorn Utzon's use of geometrical principals and mathematical knowledge allowed this building to be created with techniques never employed previously in any other structure. Inspirational ideas Utzon received from his travels and nature are evident in the opera house's design, and his creation reflects his thoughts about reaching for the heavens and being drawn to the natural world. The Sydney Opera House owes much credit to these revelations which helped Utzon with his plans, and likewise the building motivates other architects to unite nature and mathematics in their designs.

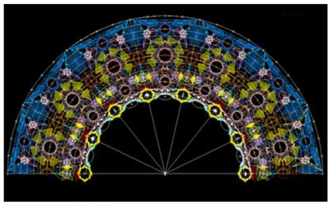
ANTONI GAUDI: LA SAGRADA FAMILIA

Perhaps one of the most fascinating examples of the manipulation of mathematics to replicate nature within architecture is the Temple Expiatori Sagrada Familia. Located in Barcelona, Spain, Antony Gaudi began one of the most ambitious construction projects in modern history. As assembly of the building began under Gaudi's leadership in 1883, the work itself has taken over one hundred years to complete, with a projected end date still decades into the future. Gaudi himself devoted forty-three years of his life to the project (The Construction Board 1). Since he regarded God as his client, he understood that while he was working on a momentous scale, there was no hurry. Instead, Temple Expiatori Sagrada Familia, or simply La Sagrada Familia, was designed with the intention of venerating God through the praise of the mathematics, logic, and functionality found in the natural world (1). Personally the work was to be the summation of all of Gaudi's past endeavors; both successes and failures were to culminate in this holy space and to find their common language through mathematics. Much like Frank Lloyd Wright and Jorn Utzon, Gaudi looked to both the philosophy behind nature as well as

specific patterns that appear and repeat themselves throughout the natural world (4). Gaudi renders, copies, manipulates, and warps the geometry found in ruled surfaces, hyperboloids, hyperbolic paraboloids, ellipsoids, helicoids, conoids, and double twisted columns, magic squares, and the natural stability of trees to praise God through mathematics.

The layout design of La Sagrada Familia initially required a critical use of symmetry in order to devise an appropriate layout of the cathedral. Gaudi's structure is to include eighteen large towers in various sizes: twelve are to represent the Apostles, four are to represent the Evangelists, the second most prominent is in ode to Mary, the Virgin Mother, and finally the largest and most central pillar is dedicated to Jesus Christ (The Accidental Mathematician 1). In order to devise an appropriate layout of these eighteen towers, Gaudi turned to the use of a frieze sort of symmetry with horizontal reflection for the grand entrance, also known as the Nativity Facade. Placed in the center of the cathedral's structure was to be the pillar representative of Jesus Christ. This was to act almost as a y-axis on which the other seventeen towers would be displayed almost evenly. The main entrance lines up on the y-axis formed by the Christ Tower. Two Evangelists towers are to be placed on either side of this axis in addition to one tower representing an Apostle. Although the vertical reflection symmetry does not continue outward into the further most boundaries of the building, Gaudi attempts to even out the weight of the remaining towers. Essentially he places the widest tower, representing the Virgin Mother, on the right of the structure. As this encompasses a large amount of breadth, a larger amount of smaller towers are placed on the left, balancing out the compositional structure and thus creating an illusion of complete symmetry.

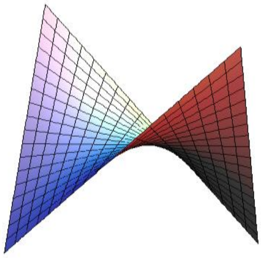
Gaudi also encountered many engineering challenges even within the conceptual phase of the cathedral's design. In that same Nativity Façade, or grand entrance, Gaudi wanted to craft pillar structures that arose from the ground organically as if they were trees (Burry 14). This would require the construction of a curved, almost parabola-shaped, titled column. The columns would not meet the ground nor the façade itself at right angles, and Gaudi was unsure of which angle would provide the enormous level of support required. To solve this problem, Gaudi became rather ingenious. He crafted a small scale hanging model of the cathedral with lax string representing the columns. He then turned this model upside down, and as gravity naturally acted on the string the necessary degree angle revealed itself organically (The Construction Board 1).



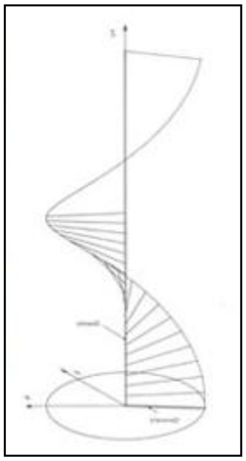
In addition to implementing mathematics in the structural formation of La Sagrada Familia, Gaudi found extensive inspiration for ornamentation, fabrication, and detail from nature's mathematical elements. The first of his ideas is the domination of the ruled surface within the cathedral. A ruled surface, in mathematical terms, is a surface containing straight lines that are "generated by the movement of one straight line that follows a particular route" (1). This gridded plane stands as the center of all of Gaudi's more complex shapes and designs (such as hyperboloids and hyperbolic paraboloids), and thus allows for easier construction and integration of these shapes. Acoustic and lighting elements with the structure are also created by the manipulation of these ruled surfaces, the most obvious of which lies in the gigantic stained glass windows. While the design for the stained glass windows are obviously geometric in its C12 rosette pattern and symmetric in vertical relief, the actual panes of colored glass are laid out in a warped ruled grid. Each panel within the windows exists on a flat plane, each appearing like its own multicolored cracked glass surface.

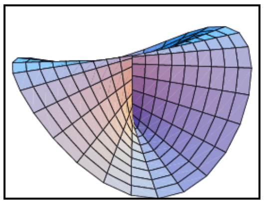
Next is the use of the hyperboloid, a “surface generated by a hyperbola which revolves around a circle of ellipses” (1). Gaudi favored this shape for lighting purposes, believing the shape symbolically represented the movement and properties of light. Gaudi seems to have taken inspiration from the Mexican pyramids at El Castillo, where the alignment of the sun’s rays is an integral portion of not only the architectural design, but the entire feeling of the space. In addition, the Sydney Opera House in generations to come will focus on the sun in a similar fashion. Both this structure and La Sagrada Familia include the reflection of light to heighten celestial associations. The vaults of the naves are pierced with hyperboloids of revolution sot that light from the attics be allowed to pass through. “Empty” or hollow hyperboloids were used for the center circles of the ceiling, where natural light could easily filter through and give the appearance of sun shining through a tree’s canopy (Burry 20). The capitals of the columns themselves culminate in a hyperboloid because the circle naturally extends it creates a funnel perfect for bearing large loads (The Construction Board 1).

Gaudi then manipulated this shape further into the hyperbolic paraboloid, “a twisted surface of a parabolic sections which is the result of a displacement of a straight line above two other lines that cross in space” (2). Generally the shape is bound by four straight lines and can be seen most easily from a twisted quadrilateral with four sides not on the same plane. Due to the fact that all generatrices of the shape rest on two straight lines, Gaudi found this shape representative of the Holy Trinity: “If one of the straight lines represented the Father, and on the opposite side the Son, the Holy Spirit was the generatrix that supported the other two and joined them permanently” (4). In order to construct the tree-like canopy of the ceiling structure, an interplay of the vaulted ceiling and the windows is conducted through intersections of hyperboloids linked with paraboloids. These shapes can easily integrate into each other because the surfaces of both of these shapes are formed by two sheaves of inclined straight lines, and thus share a similar vocabulary based on the ruled surface (The Accidental Mathematician 1). Lastly, the central towers dedicated to Jesus Christ, Virgin Mary, and the four Evangelists are formed by vertically stretched hyperbolic paraboloids.

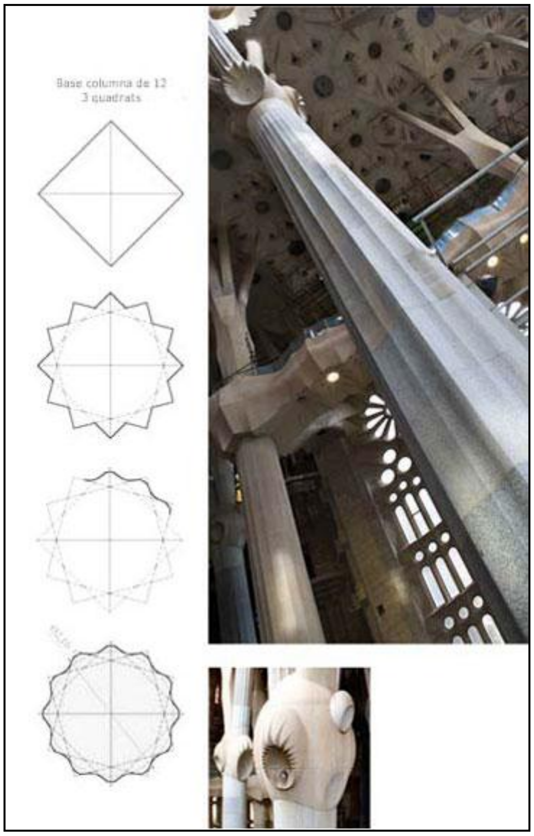


Other geometric shapes used extensively by Gaudi are the helicod and conoid. A helicod is a ruled surface generated by a straight line that revolves according to a spiral around a vertical axis, or essentially a spiral staircase without the stairs. For Gaudi this shape symbolized the natural ascension of the earth to the heavenly realm (*"Sagrada Familia "Church of the Holy Family"*. 1). Thus it goes without saying that the staircases within La Sagrada Familia are mostly helicods. What is more interesting, though, is how these staircases also form fractals when looked at from above. In particular, the staircases leading up the central main towers create a fractal spiral design reminiscent of a spiral seashell. It is this use of fractals that will inspire future architectures like





Frank Lloyd Wright to include them in his own work. Wright, however, takes many of these principles further, such as in the Palmer house, where the floor plan of the structure, layout of the rooms, and various details are displayed in fractal formations. In addition, the twisted columns that make up the bases of the "tree trunk" structures are developed from the geometry behind the helicod. It was the idea of the tree structure that serves as Gaudi's largest inspiration from nature. Within trees, nature crafted an "orderly hierarchical structure that provides optimal support for a beautiful vault of leaves" (The Construction Board 1). In this case, nature not only inspired aesthetic designs but structural inspirations as well. In trying to replicate this natural structure of a tree, Gaudi calculated the weight of each part of the roof and then the center of gravity for each portion. He then replaces the arches with columns and "gave them the necessary inclination to take the exact direction of the descent of loads; in that way, working with centered compression, he could make the structure extremely slender, which is what makes it so extraordinary" (The Accidental Mathematician 1). The conoid is a ruled surface formed by a straight line, which is displaced above another straight line and then above a curve (2). To formally construct a conoid, "a sinusoidal guideline is traced on the ground and a straight bar placed at a certain height as a



second guideline. Resting on these two guidelines, one a straight line and one a curve, are the straight lines that generate the surface, which are the whole group of tensed strings linked to the upper bar that seek the line marked below to act as a guideline for raising the undulating wall" (The Construction Board 1). In particular, the roof of the attached religious school is constructed in this conoid structure. Here the wooden beams are the generatrices that rest on the sinusoidal profile of the facades, with a central interior main beam on the other.

One of the last geometric shapes used by Gaudi to replicate nature is the ellipsoid. As a solid shape in which all the flat sections are ellipses, he found that adding or subtracting different ellipsoids from one another created a multitude of interesting shapes and designs (Burry 15). Within the context of La Sagrada Familia, the shape of the ellipsoid represents the "knots" of the "trees" and thus the capitals of the columns. The columns themselves are works of engineering genius. In order to replicate the grooved look and feel of tree bark, Gaudi crafted double twisted columns. These columns begin with either a regular or starred polygon at the base with either straight of parabolic sides. As the column rises, the sides are transformed into different sections with increasing numbers of vertices. It serves as an intersection of two helicoidally columns with the same base, but opposite twists. The column culminates in the previously mentioned ellipsoid capitals, and the smaller "branches" which rise upwards from this capital are also double twisted, but with different polygons at the base (25).

Gaudi does apply some mathematics beyond the large domination of geometric forms, however. On the Passion façade of La Sagrada Familia is the inclusion of a magic square. Although known to Chinese mathematicians as early as 650 BC and Arab mathematicians in the seventh century, India, Egypt, Persia, and Europe all have traditions of the magic square. These cultures used magic squares both for mathematics and as talismans believed to have astrological and divinatory qualities, to ensure longevity and to ward off disease. (The Construction Board 1). Specifically the La Sagrada Familia magic square is a four by four construction. It is not a "regular" magic square, however, as it repeats the numbers fourteen and ten and excludes twelve and sixteen. When the rows, columns, and diagonals of this square are added up, they equal thirty-three-the age of Jesus Christ at the time of the Passion (The Accidental Mathematician 1).

If nature and mathematics ever were to get married, La Sagrada Familia would be the place for them to do it in. Even though this architectural wonder is not yet finished, the designs that have already been put into place in this building continue to inspire and amaze people worldwide. Like the tree-replicated columns inside the church, this building will continue to grow and expand as we move ahead into the future.

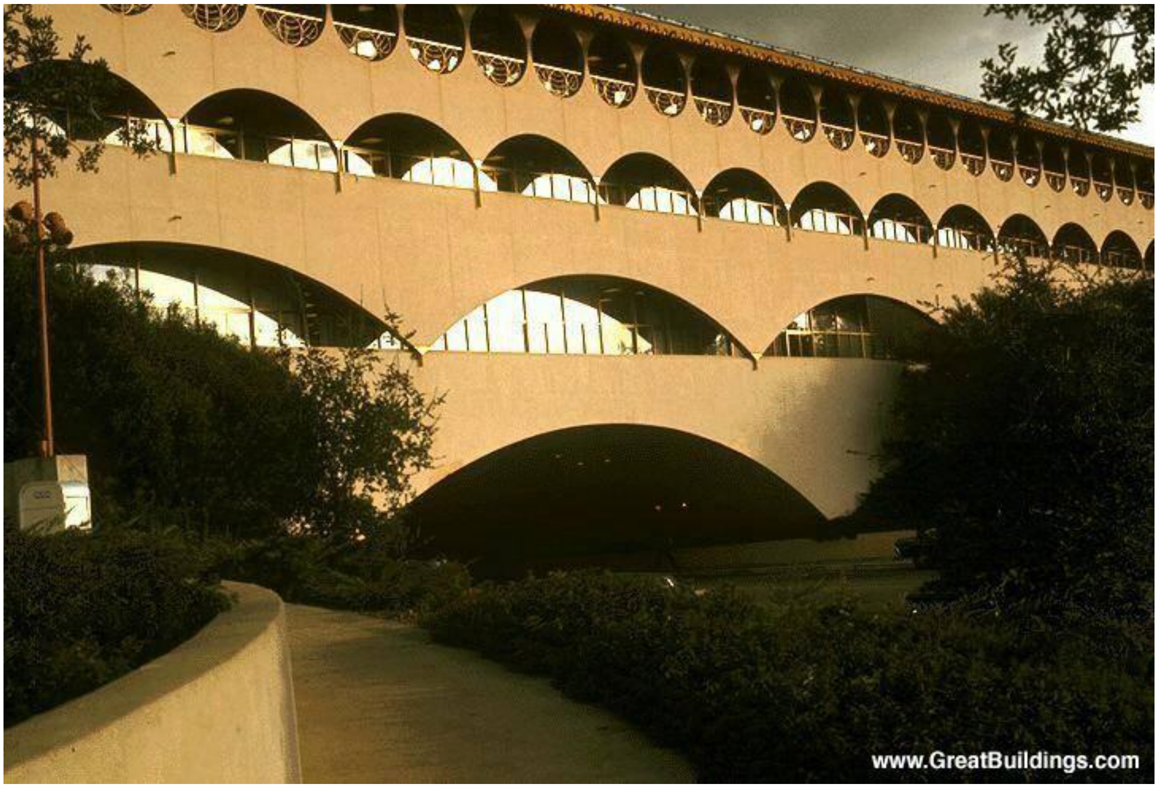
FRANK LLOYD WRIGHT: AMERICAN ARCHITECT

In 20th century America, both domestic and commercial architecture was being completely redefined by a single man: Frank Lloyd Wright. Wright, who was born in 1867, completed over five hundred structures and designed many more. He was also a prominent advocate of organic architecture, which is the philosophy that architecture should be designed in a way that it promotes harmony between human habitation and the natural world. This calls for designs that are aware and sympathetic to their immediate surroundings and eventually become an organic and essential element of the landscape. The most famous example of organic architecture is Fallingwater, a house designed by Wright himself for the owner of the Kauffman Department stores. Interestingly enough, another strong believer in organic architecture was Antoni Gaudi, the creator of the Sagrada Familia. Alongside Fallingwater, the Guggenheim museum in New York City comprises his two most famous works, however for the purpose of this project I chose to look at two of his lesser known buildings. Both of these buildings, as do his others, embody a natural appearance and feel, while at the same time incorporating mathematical and geometrical elements to further this cause.

The first structure is the Marin County Civic Center in Marin County, San Rafael, California. The design for this building was completed in 1957, however the actual building was not completed until the mid 1960's. This means that just like Gaudi, Wright, who passed away in 1959, did not live long enough to see some his greatest ideas and designs come to fruition. The actual materials used in the design were typical to Wright. He used ample amounts of concrete in the construction of his buildings. Concrete was an imperative element to Wright's adherence and extension of organic architecture due to its ultra-malleable nature and natural appearance.

The civic center is a superlative example of Wright's use of complex and unusual mathematical concepts in his designs. Foremost his use of fractal geometry is immediately noticeable, and is an element of his designs that is not unique to this building or commercial buildings in general. The façade of the civic center is multiple series of arches layered on top of each other, with the distance at the base of the arches being halved as you move up each successive layer. Because of the relationship between the layers of archways, this design actually represents a fractal as the bottom arch is divided to make smaller arches on the second level.

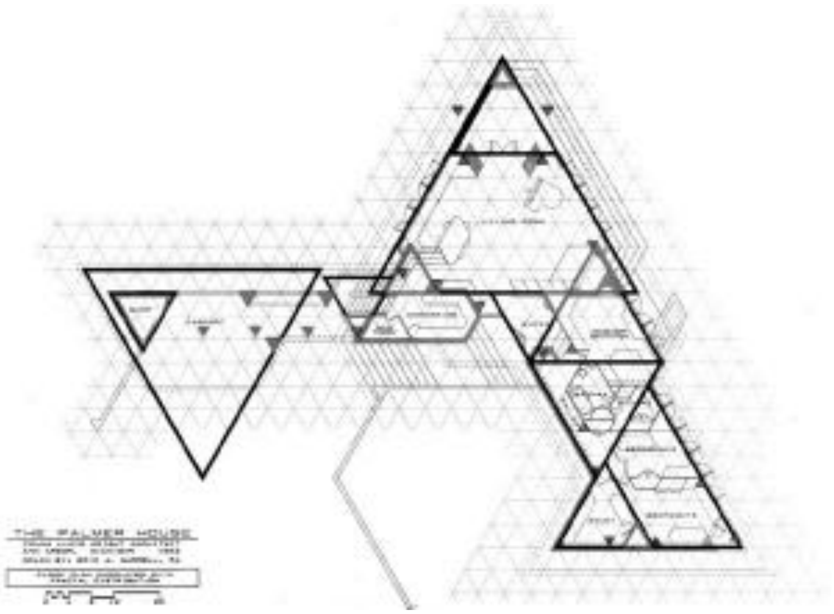
Those arches on the second level are subsequently divided as well to make yet smaller arches for the third level. The continuation of this pattern is definitional of a fractal. Wright's use of the super and sub structures is an ingenious and unique use of fractal geometry in architecture (Sala).



As previously mentioned, Wright was a huge advocate for organic architecture and naturalism in his designs, principles that were consistently applied in his architecture. One of these principles was that each building must have a certain interplay with its surroundings. Because of this he gave the Marin Civic Center an unusual free flowing design while incorporating a plethora of unique and simple geometrical features that in unison create a very natural essence. From above, the building looks as if he wanted it to resemble a stream or waterfall, a feature that he toyed with much throughout his career (Hilderbrand).

Wright's place in the architectural world was nothing short of phenomenal. Even though his designs cannot be traced to any one influence and are considered completely unique, his use of fractals and other mathematics has existed in architecture for millennia. Also, his conceptions of the relationship his designs interact with nature were not entirely his own, and have been pondered by many architects.

The second structure of Wright's I looked at was one of his last domestic masterpieces. The Palmer House was built in 1950 in Ann Arbor, Michigan for William and Mary Palmer. Space plays a central role in the design of this house, and it is focused around a central room. Not only was Wright the architect for this house but he also designed and situated all the furniture and fixtures, a common practice of his. The floor plan of this house also uses fractal geometry and the entire structure is essentially a single triangle that is dilated, rotated, and translated multiple times to form the rooms and walls.



Even though this type of fractal use is only apparent in the blueprints, the triangle shape is consistently used in all elements of the home. The triangle is the primary theme on all scales of the design and is used in fixture such as floor times as well as sub-structures such as vaulted ceilings. We continue to see Wright use simple geometrical shapes, synthesized to create an utterly natural feel (www.flwpalmerhouse.com).

The house also maintains a unique relationship with its surroundings, conforming to the hill it is set on rather than altering the hill meet its purpose. This is typical of Wright's houses and is in accordance with his foremost principle of integrating architecture and nature into an organic whole.

CONCLUSION

Mathematics in symmetry, geometry, and proportions are continuously found in the natural world. Inspired by the beauty displayed in the natural world, architects use mathematics as a way to replicate nature in all its glory through the designs of their buildings. Venerations of nature are integral means in the designs, ranging from ancient civilizations such as the Mayans to modern day architects such as Frank Lloyd Wright. Mathematics from the cosmos are also used to help highlight and enhance natural beauty. Sunlight, stars, and the galaxy are used when thinking of the shape, texture, and overall layout of the structures. Because architects worldwide are inspired by nature and mathematics when creating their buildings, mathematics transcends cultural boundaries, allowing nature to be the common language.

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HOMEWORK

1. Locate a picture of a building you have encountered. It can be a personal picture you took at home, on campus or on vacation, or it can be a picture found on the internet. List the mathematical qualities of the building's design. For example, are there any patterns, geometric shapes, or magic squares present?
2. Nature has inspired all of the buildings in our project. How has and/or does nature inspire you? Write a few sentences about it.

POSSIBLE HOMEWORK SOLUTIONS

1. The building I selected is the Hearst Magazine Building. This structure has many shapes in it including rectangles and triangles. The triangle-shaped windows are reflected and rotated (a frieze pattern). The rectangular windows on the bottom of the building are translated horizontally and vertically. The rectangles may also be golden rectangles, but without the measurements of the building it is difficult to verify this. Photograph from: http://wirednewyork.com/real_estate/hearst_magazine_building/images/hearst_building.jpg

2. Nature inspires me both in my writing and in my photography. My favorite photographs to take are close-ups of flowers, panoramic views of the mountains, and pictures of animals (especially my guinea pigs!). I also love to sit on the shores of lakes and ponds and practice my creative writing; I am often inspired to write poetry after spending some time at my family's lake house. Nature inspires me to create works of art just like many architects are inspired to create architectural masterpieces that replicate natural beauty.